Harel’s DNR-Based Instruction

- What is DNR-Based Instruction?
- Why use DNR-Based Instruction?

What is DNR-based Instruction?

- A Theoretical Model to Guide Instruction
- 4 Underlying Premises
- 3 Pedagogical Principles

What are the 4 Premises?

Subjectivity Premise

Any observations human claim to have made are due to the attribution of their mental structure to their environment.” (Harel, 2007, p. 265)

Implications:
- What you show isn’t necessarily what you student see.
- What you tell isn’t necessarily what they hear.
- What you teach isn’t necessarily what they learn.
- We have to consider students’ existing schemes.

Knowledge Development Premise

“The process of knowing is developmental in the sense that it proceeds through a continual tension between accommodation and assimilation.” (Harel, 2007, p. 265)

Implications:
- Students do not learn by absorbing knowledge, nor by merely replacing their existing conceptions.
- Students learn by refining their existing conceptions.

Teaching Premise

“Construction of scientific knowledge is not spontaneous. There will always be a difference between what one can do under expert guidance or in collaboration with more capable peers and what one can do without guidance.” (Harel, 2007, p. 265)

Implications:
- Teacher facilitates learning by providing opportunities for students to collaborate and to share their reasoning.
- Teacher plays the role of the coach.

Mathematics Epistemology Premise

“Knowledge of mathematics consists of all the ways of understanding and ways of thinking that have evolved throughout the history of mathematics.” (Harel, 2007, p. 265)

Implication:
- Both WoU and WoT should be included as learning objectives.
What is the Difference between WoU and WoT?

First consider what it takes to know and do mathematics.

**Knowing & Doing**
- Mathematics as a Discipline
  - Interconnected Knowledge of Mathematics
  - Mathematical Habits of Mind

What is the Difference between WoU and WoT?

"The first subset [of mathematics] is a collection, or structure, of structures consisting of particular axioms, definitions, theorems, proofs, problems, and solutions. This subset consists of all the institutionalized ways of understanding in mathematics throughout history."

(Harel, in press)

What is the Difference between WoU and WoT?

"The second subset consists of all the ways of thinking, which characterize the mental acts whose products comprise the first set."

(Harel & Sowder, 1998)

What is the Difference between WoU and WoT?

Why is sum of two odd numbers even?

"My teacher said so."
"It's stated in the text book."

Authoritative proof scheme

1 + 3 = 4 even
1 + 5 = 6 even
7 + 9 = 16 even
13 + 25 = 47 even

odd + odd = (even + 1) + (even + 1)

1 + 1 become even.
Even plus even is even.

Empirical proof scheme

odd + odd = even + even

Deductive proof scheme

(Harel & Sowder, 1998)
**Mathematical Knowledge for Teachers**

**Pedagogical Content Knowledge**

**Subject-matter Knowledge**

**Curricular Knowledge**

**Ways of Understanding**

**Ways of Thinking**

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**The Duality Principle**

“Students develop ways of thinking only through the construction of ways of understanding, and the ways of understanding they produce are determined by the ways of thinking they possess.”

(Harel, 2007, p.272)

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**The Necessity Principle**

“Students are most likely to learn when they see a need for what we intend to teach them, where by ‘need’ is meant intellectual need, not social or economic need.”

(Harel, 2007, p. 274)

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**The Repeated-reasoning Principle**

“Students must practice reasoning in order to internalize, organize, and retain ways of understanding and ways of thinking.”

(Harel, 2007, p. 275)

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**Is the Most-square Problem in accordance with Harel’s Pedagogical Principles?**

A new housing subdivision offers rectangular lots of three different sizes:

- A. 75 feet by 114 feet
- B. 455 feet by 508 feet
- C. 185 feet by 245 feet

If you were to view these lots from above, which would appear most square?

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**Why Use DNR-Based Instruction?**

- Allows students to experience the shortcomings of their existing WoU and WoT
- Intrigues students
Question: Which method should I use?

- Allows students to experience the shortcomings of their existing WoU and WoT
- Intrigues students
- Promotes attending to meaning of numbers/symbols

Why Use DNR-Based Instruction?

Attending to Meaning

Question: Can you show what 39 represents in the diagram?

- Difference = Length – Width
- B 114 feet
   - 75
   - 39

Attending to Meaning

Question: What does the ratio 1.12 actually represents?

- 508 is about 1.12 times of 455
- B 114 feet
   - 75
   - 39

Explaining & Justifying

Question: Why is one method is better than the other?

- Difference = Length – Width
- Ratio = Length / Width

- Explaining
- Justifying

Intriguing Students
Question: Why is the difference method not good?

Question: Why is the ratio method better?

Characteristics of a Good Task
- Builds on students existing WoU and WoT
- Provokes an intellectual need for a concept
- Requires students to explain and justify
- Promotes making connections

Multiple Ways of Understanding Ratio
- Ratio as a multiplicative comparison of 2 quantities (e.g. Length of Rect. B is 1.12 times of Width of Rect. B) (e.g. Area of Rect. B is 1.12 times of Area of Square)
- Ratio as a measure of “squareness” (e.g. Rectangle B has a measure of 1.12 for squareness)
- Unit ratio as a means for comparing measures

Meaning of Unit Ratio

Ratio = 1.52

Ratio = 1.12

Ratio = 1.32
The Duality Principle

What WoU can this activity potentially foster?
- Difference as an additive comparison
- Ratio as a multiplicative comparison
- Ratio as a means to measure “squareness”
- Unit ratio strategy for comparing ratios

What WoT can this activity potentially foster?
- Referential symbolic reasoning (attend to meaning)
- Multiple ways of understanding a concept is advantages
- Changing the form without changing the attribute (e.g. changing the size without changing the shape)