Constrained Optimization Framework for Joint Inversion of Geophysical Datasets

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I. Abstract
We implement a joint inversion least-squares (LSQ) algorithm to characterize 1D Earth structure. We use two geophysical datasets, namely Receiver Function (RF) and Surface Wave (SW) dispersion velocity observations, with a choice of an optimization method. Two-dimensional Inverse Decomposition (TVID) or Primal-Dual Interior-Point (PDP) methods are used to solve a regularized constrained minimization problem, respectively. Both techniques include bounds into the model (layered shear velocity) with a different methodology. We conduct a numerical experimentation with four synthetic problems, and find that the PDP method provides a more robust model in terms of satisfying geophysical constraints, accuracy, and efficiency than the TVID approach.

II. Introduction
In geophysics joint inversion involves the simultaneous optimization of several objective functions to find a model that fits all datasets at once. One approach is to add weights to the objective functions for each dataset. Previous work includes resistivity and magnetotelluric data, Capano et al. (2017), seismic travel times and gravity data (Lee et al., 1992), and RF and SW dispersion data (Jaba et al. 2006). Our contribution in this framework is to integrate two datasets that incorporates explicitly constraints over the model. The success of the joint inversion is based on:

1. Each dataset provides information of the same propagating medium (complementarily).
2. The combination of the datasets may bridge resolution gaps (complementarily).

Advantages: The complementarily characteristics impose better physical constraints that increases the resolution of the model.

Disadvantages: The technique involves highly nonlinear functions that may lead to infeasible models (local minima). The selection of weights/regularization parameters is an open research question.

- Forward problem: Given a velocity model v = (v1, v2, v3) we evaluate a linear operator F as the group velocity to predict the Earth response, where m is the number of observations, n is the number of layers predefined and m>n.

- Inverse problem: Given datasets y, find the unknown model x such that F(x) = y.

III. Unconstrained Joint Inversion Formulation (cont’d)
The inverse problem is posed as a nonlinear constrained LSQ problem (1) usually 8-conditioned as follows:

\[ \min_{x} \left\{ \frac{1}{2} \| F(x) - y \|^2 + \frac{1}{2} \| L(x) - \zeta \|^2 \right\} \]

where \( F \) is the forward operator, \( y \) is the observed data, \( L \) is the measurement operator, \( \zeta \) is the measurement noise, and \( x \) is the unknown model. The problem is a constrained least-squares problem with the first term being a data misfit and the second term is a regularization term used to stabilize the inversion using the vector \( R \).

Receiver Function (RF): The teleseismic RF technique is used to determine the Earth response near a recording station to an incoming P-wave. In summary RF techniques:

- resolve impedance contrasts of deep (discontinuous) layers.
- usually does not provide good average of shear wave velocity.
- provide shear-wave velocity contrasts in layered structures, and good measurement of crustal thickness.

Surface Waves Dispersion Velocities (SW): (Figs. 1c) Surface waves, Love and Rayleigh waves, have energy concentrated near Earth surface. In summary SW:

- do not resolve layers of high impedance contrast of deep layers.
- provide average of absolute shear wave velocity.
- provide shear-wave velocity contrasts in layered structures.

IV. Datasets (cont’d)

V. Constrained Joint Inversion Formulation
We propose a different strategy to solve (2) by controlling directly the constraints in the inversion problem by means of the constrained formulation:

\[ \min_{x} \left\{ \frac{1}{2} \| F(x) - y \|^2 + \frac{1}{2} \| L(x) - \zeta \|^2 \right\} \quad \text{subject to } g(x) \leq 0 \]

where \( g(x) \) is the natural constraint of the problem, \( y \) is the observed data, \( L \) is the measurement operator, \( \zeta \) is the measurement noise, and \( x \) is the unknown model.

VI. Numerical Experimentation (cont’d)

- Two algorithms were compared and implemented for all the synthetic models:
  - Joint Inversion Algorithm
  - PDP Algorithm

VII. Conclusions and Future Work
- As shown in Figs. 3a-3a, PDP outperforms or is as good as the traditional unconstrained regularized inversion that uses TSOV, when solving the constrained joint inversion formulation of (1).
- The PDP algorithm is a better approximation to the true models without the regularization term needed for convergence of the TSOV strategy.
- The computational complexity in terms of floating point operations is about 18 times more for TSOV than for PDP.
- Future work includes the implementation of an iterative method in PDP for solving system (5), and
- To implement the PDP to jointly invert large scale real geophysical data, i.e., seismic data recorded for the Rio Grande Rift region.

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