3-D Constrained Joint Inversion for Crustal and Mantle Velocity Structure in the Rio Grande Rift (RGR)

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3 Crustal and (Upper) Mantle Velocities of the RGR
Cyber-ShARE Center of Excellence

We are not students of some subject matter, but students of problems. And problems may cut right across the borders of any subject matter or discipline.

Karl Popper
Cyber-ShARE Center of Excellence

- Created in 2007 by a NSF grant to bring together experts in computer science, applied mathematics, Earth science, and environmental science.
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- **Geoscience** - Integrated analysis for development of 3-D models of Earth structure.

- Our interdisciplinary team:
  - Applied Mathematics (constrained joint inversion).
  - Computer Science (programming support and visualization).
  - Geological Sciences (data processing and interpretation).
Motivation & Background

- Is the RGR still active? What is the role of the mantle in rift formation?

Advantage: complementary data improves final (velocity) model resolution.

Disadvantages: data weighting (level of influence), presence of spurious solutions, and selection of regularization parameters.
Motivation & Background

How can multiple geophysical datasets with different sensitivity and resolution domains be integrated to characterize Earth structure?
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Our choice $\rightarrow$ Joint inversion scheme [Julia et. al. 2000; Moorkamp et. al., 2010]

- Advantage: complementary data improves final (velocity) model resolution.
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Our approach: A constrained optimization least squares (LS) formulation for joint inversion of geophysical data.
Our goal: Use joint inversion of observed ground motion measurements, \( y = (y_1, \ldots, y_m) \), to estimate \( S\)-wave velocities \( x = (x_1, \ldots, x_n) \).
Joint Inversion Schemes

Unconstrained Joint Inversion

\[
\min_x \frac{1}{2} \|F(x) - y\|_W^2 + \frac{\lambda}{2} \|Lx\|_2^2, \tag{1}
\]

where \( F(x) = \begin{bmatrix} F^{SW}(x) \\ F^{RF}(x) \end{bmatrix} \in \mathbb{R}^m \),

\[
y = \begin{bmatrix} y^{SW} \\ y^{RF} \end{bmatrix} \in \mathbb{R}^m, \quad \lambda \text{ is a regularization parameter and } L \text{ a discrete derivative matrix.}
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Constrained Joint Inversion

\[
\min_x \frac{1}{2} \| F(x) - y \|_W^2 \quad \text{s. t.} \quad g(x) \geq 0,
\]

where

\[
g(x) = \begin{bmatrix}
x - c_{min} \\
c_{max} - x \\
\gamma \frac{1}{n} - \frac{1}{2} \| Lx \|_2^2
\end{bmatrix},
\]

introduces a structural constraint over the model \( x \) with \( \gamma \in (0, c_{max} - c_{min}) \).
Joint Inversion Overview

Joint Inversion Schemes

- **Unconstrained Joint Inversion**

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- **Constrained Joint Inversion**

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\]

introduces a structural constraint over the model \( x \) with \( \gamma \in (0, c_{max} - c_{min}) \).

- Problems (1) - (2) are linearized and solved iteratively.
Unconstrained Joint Inversion

- Robust algorithms for solving (1), i.e. **TSVD**.
- **Key idea**: To penalize constraint violations.
- Requires computation of adequate **regularization** parameters.

Constrained Joint Inversion

Apply techniques from the optimization field for joint inversion.

Powerful algorithms for solving large-scale problems.

**Key idea**: To keep iterates feasible with inequality constraints.

Requires a good starting model and robust equation solvers.

"Constrained Optimization Framework for Joint Inversion of Geophysical Data Sets", A. Sosa et al. GJI (To be submitted)

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Unconstrained vs Constrained Joint Inversion Schemes

Unconstrained Joint Inversion

- Robust algorithms for solving (1), i.e. TSVD.
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Constrained Joint Inversion

- Apply techniques from the optimization field for joint inversion.
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Primal-Dual Interior Point Methods \cite{ElBakry1996}

The Lagrangian function associated to (2)

\[ \ell(x, z) = \frac{1}{2} \| F'(x_k)x + r(x_k) \|^2 - g(x)^T z, \quad (g(x), z) > 0, \quad (3) \]
Primal-Dual Interior Point Methods [El Bakry et. al. 1996]

The *Lagrangian* function associated to (2)

\[
\ell(x, z) = \frac{1}{2} \| F'(x_k)x + r(x_k) \|^2 - g(x)^T z, \ (g(x), z) > 0,
\]

leads to

\[
\begin{bmatrix}
-F'(x_k)^T F'(x_k) + z_{2n+1} L^T L & \nabla g(x)^T \\
\nabla g(x) & Z^{-1} G(x)
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta z
\end{bmatrix} =
\begin{bmatrix}
-\nabla x \ell(x, z) \\
\mu Z^{-1} e - g(x)
\end{bmatrix}
\]

\[
Z = \text{diag}(z), \quad G(x) = \text{diag}(g(x))
\]
Joint Inversion by Using PDIP for RGR Seismic Data

We aim to characterize crustal and mantle velocity structure of the RGR.¹

- Data for the RGR from 147 stations of the USArray survey and LA RISTRA experiment.
  - 147 Rayleigh and Love group velocity dispersion curves (Data provided by Dr. Robert B. Herrmann http://www.eas.slu.edu/eqc/eqc_research/NATOMO).
  - Stacked 434 RF in ray parameter bins.

Figure: Topography, tectonic provinces, and stations used (mostly from US Array).

¹Lennox Thompson et. al. in preparation.
SW and RF for Station 427A

- Observed Love
- Estimated Love
- Observed Rayleigh
- Estimated Rayleigh

$p = 0.0446$
Crustal and (Upper) Mantle Velocities of the RGR

SW and RF for Station 427A

Joint Inversion for the RGR

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Crustal and (Upper) Mantle Velocities of the RGR

SW and RF for Station 427A

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Joint Inversion for the RGR

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Sw and RF for Station 427A

![Graph showing group velocity and amplitude over period and time](Image)

- Observed Love
- Estimated Love
- Observed Rayleigh
- Estimated Rayleigh

\[ p = 0.0763 \]
S-wave Velocity ($V_s$) models for the RGR

Constrained joint inversion and kriging interpolation used to create 3-D velocity models of the RGR.²

²A. Sosa et. al. in preparation.
$S$-wave Velocity ($V_s$) models for the RGR

Constrained joint inversion of independent data per station provide us with 1-D velocity models.

![Diagram of shear velocity vs depth for the RGR models.](image-url)
**S-wave Velocity ($V_s$) models for the RGR**

A Kriging interpolation scheme incorporates uncertainty into the 1-D models and smooths out our sparse distributed data.
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S-wave Velocity ($V_s$) models for the RGR

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2-D $V_s$ Cross-sections for RGR

Cross Section at 32 degrees

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2-D $V_s$ Cross-sections for RGR
3-D $V_s$ Velocity Models
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3-D $V_s$ Velocity Models
LA RISTRA Experiment Comparison [Wilson et. al. 2004]
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Conclusions

- We presented a robust approach to introduce *a priori* information for joint inversion.
- 1-D velocity profiles used as input for our Kriging interpolation scheme allow us to create 3-D models with smoother varying results than previously known.
- Our joint inversion results suggest the RGR has a mantle signature that may extend to the southeast.
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- 1-D velocity profiles used as input for our Kriging interpolation scheme allow us to create 3-D models with smoother varying results than previously known.

- Our joint inversion results suggest the RGR has a mantle signature that may extend to the southeast.

**Future work will include:**

- Finalize 3-D model visualization of the RGR, and include other compatible datasets, e.g. gravity and delay time data.
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