Some delicate fabrics are natural silks, made of protein and biodegradable. It would be of great benefit to be able to assess the delicacy of the fabric before making decisions about displaying it in a museum. Chemical analysis might give some evidence about the brittle nature of a fabric. Bio-chemical data in the form of a ratio of the amount of certain amino acids in the fibers were acquired from the linings of sixteen 19th and early 20th century Japanese kimonos, as well as the breaking stress of the fabrics.

1. Using the data from the Japanese kimonos, construct the least squares best fit line for breaking stress (“tenacity”) as a function of the amino acid ratio.

   a) What is the equation of the least squares best fit line?

   b) Graph the least squares best fit line on the scatter plot that appears on the next page.

   c) Approximately what proportion of the variability in breaking stress is explained by the amino acid ratio?
2. The theory of fiber strength suggests that the relationship between fiber breaking stress and amino acid ratio is logarithmic, i.e. 
\[ \hat{S} = \alpha + \beta \log(R) \], where \( S \) is the breaking stress and \( R \) the amino acid ratio. Perform the appropriate transformation of variable(s) and fit this logarithmic model to the data.

(a) What is the resulting best fit line using this model?

(b) For an amino acid ratio of \( R = 1.5 \), what is the predicted breaking strength?
7. The early identification of children who might be at risk for future learning difficulties is a high priority with school districts. Research has consistently shown that the earlier the intervention is initiated the more effective it is. The proficiency standard for the State of Iowa is achievement at the 40th percentile on a nationally standardized test. The best fit curve shown below is from a logistic regression relating the estimated probability of achievement (at the 40th percentile) in 8th grade to the student’s 3rd grade score on the test.

\[
y = \frac{e^{-16.403 + 0.1044x}}{1 + e^{-16.403 + 0.1044x}}.
\]

(a) Calculate the estimated probability of success for a student who gets a standard score of 160 in 3rd grade?

(b) School district officials would like to identify the students whose estimated probability of success is 0.10 or less. Approximately what standard score should they use for identification?