Chapter 12

12.3 How much would the average lead time be reduced?

*Given constant service times, Queuing Model 2 (cf. Chapter 9, page 350) applies.*

\[
\lambda = 90 \text{ units/hour}, \ \mu_1 = 95 \text{ units/hour}, \ \mu_2 = 105 \text{ units/hour}
\]

\[
\bar{t}_s = \bar{t}_i + \frac{1}{\mu} = \frac{\lambda}{2\mu(\mu - \lambda)} + \frac{1}{\mu}
\]

\[
\bar{t}_{s1} = \frac{90}{2(95)(95 - 90)} + \frac{1}{95} = \frac{90}{950} + \frac{1}{95} = 0.094737 + 0.010526 = 0.10526 \text{ hour or 6.316 minutes}
\]

\[
\bar{t}_{s2} = \frac{90}{2(105)(105 - 90)} + \frac{1}{105} = \frac{90}{3150} + \frac{1}{105} = 0.028571 + 0.009524 = 0.0381 \text{ hour or 2.286 minutes}
\]

Lead time would be reduced from 6.316 minutes to 2.286 minutes, or a reduction of slightly more than 4 minutes.

12.4 In Problem 3, how much would the average WIP be reduced with the improvement in lead time?

\[
\lambda = 90 \text{ units/hour}, \ \mu_1 = 95 \text{ units/hour}, \ \mu_2 = 105 \text{ units/hour}
\]

\[
\bar{n}_s = \bar{n}_i + \frac{\lambda}{\mu} = \frac{\lambda^2}{2\mu(\mu - \lambda)} + \frac{\lambda}{\mu}
\]

\[
\bar{n}_{s1} = \frac{90^2}{2(95)(95 - 90)} + \frac{90}{95} = \frac{8100}{950} + \frac{90}{95} = 8.5263 + 0.9474 = 9.474 \text{ units}
\]

\[
\bar{n}_{s2} = \frac{90^2}{2(105)(105 - 90)} + \frac{90}{105} = \frac{8100}{3150} + \frac{90}{105} = 2.5714 + 0.8571 = 3.429 \text{ units}
\]

WIP would be reduced from 9.474 to 3.429 units, or 6.045 units—a reduction of about 64 percent.
12.6 Use the *POM Software Library*, what is the average WIP at the operation? What is the average lead time at the operation?

Here is computer printout from the *POM Software Library*:

MODEL: Multiple Channels

arrival Rate (lambda) = 70
Service Rate (mu) = 25
Number of Channels = 3

Average Number of Units in Waiting Line = 12.2735
Average Number of Units in System = 15.0735
Average Waiting Time in Line = 0.1753
Average Time in System = 0.2153
Probability of Idle System = 0.0160

Probability of 1 units in the system = 0.0447
Probability of 2 units in the system = 0.0626
Probability of 3 units in the system = 0.0584
Probability of 4 units in the system = 0.0545
Probability of 5 units in the system = 0.0509
Probability of 6 units in the system = 0.0475
Probability of 7 units in the system = 0.0444
Probability of 8 units in the system = 0.0414
Probability of 9 units in the system = 0.0386
Probability of 10 units in the system = 0.0361
Probability of 11 units in the system = 0.0337
Probability of 12 units in the system = 0.0314
Probability of 13 units in the system = 0.0293
Probability of 14 units in the system = 0.0274
Probability of 15 units in the system = 0.0255
Probability of 16 units in the system = 0.0238
Probability of 17 units in the system = 0.0222
Probability of 18 units in the system = 0.0208
Probability of 19 units in the system = 0.0194
Probability of 20 units in the system = 0.0181
Probability of 21 units in the system = 0.0169
Probability of 22 units in the system = 0.0158
Probability of 23 units in the system = 0.0147
Probability of 24 units in the system = 0.0137
Probability of 25 units in the system = 0.0128
Probability of 26 units in the system = 0.0120
Probability of 27 units in the system = 0.0112
Probability of 28 units in the system = 0.0104

The average WIP (Average Number of Units in System) is 15.0735 products and the average lead time is .2153 hour (Average Time in System) or 12.92 minutes.
Using STORM:

**STORM DATA SET LISTING**

PROBLEM DESCRIPTION PARAMETERS
Title : Problem 12.6
Number of independent queueing problems : 1

DETAILED PROBLEM DATA LISTING FOR
Problem 12.6

QUEUE 1
# SERVERS   3
SOURCE POP  INF
ARR RATE    70
SERV DIST   EXP
SERV TIME   0.04
SERV STD    .
WAIT CAP    .
# CUSTOMERS .
WAIT COST   .
COST/SERV   .
LOSTCUST C .

**STORM Output**

Problem 12.6
QUEUE 1 : M / M / c
QUEUE STATISTICS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of identical servers</td>
<td>3</td>
</tr>
<tr>
<td>Mean arrival rate</td>
<td>70.0000</td>
</tr>
<tr>
<td>Mean service rate per server</td>
<td>25.0000</td>
</tr>
<tr>
<td>Mean server utilization (%)</td>
<td>93.3333</td>
</tr>
<tr>
<td>Expected number of customers in queue</td>
<td>12.2735</td>
</tr>
<tr>
<td>Expected number of customers in system</td>
<td>15.0735</td>
</tr>
<tr>
<td>Probability that a customer must wait</td>
<td>0.8767</td>
</tr>
<tr>
<td>Expected time in the queue</td>
<td>0.1753</td>
</tr>
<tr>
<td>Expected time in the system</td>
<td>0.2153</td>
</tr>
</tbody>
</table>

The average WIP (Expected number of customers in system) is 15.0735 units and the average lead time is .2153 hour (Expected time in the system).
12.10

a. What is the current average labor cost per part, including changeovers?

\[
\text{Cost/part} = \frac{\text{production cost} + \text{setup cost}}{\#\text{parts}}
\]

\[
\text{Cost/part} = \frac{[2.75(30) + 25](15.75/60)}{30}
\]

\[= \$0.941 \text{ per part}\]

b. If the changeover time could be reduced to 10 minutes, how much labor cost per part would be saved using the current batch size of 30 units?

\[
\text{Cost/part} = \frac{[2.75(30) + 10](15.75/60)}{30}
\]

\[= \$0.809 \text{ per part}\]

The savings per part is \$0.941 - \$0.809 = \$0.132

c. If the changeover time could be reduced to 10 minutes, how much could the batch size be reduced in order to achieve the current average labor cost per unit?

Let \(Q\) = new batch size (in \# of units).

Total cost = setup cost + labor cost

\[
= \left(10 \text{ minutes} \times \frac{15.75 / \text{hour}}{60 \text{ minutes/hour}}\right) + \left(2.75 \text{ minutes/unit} \times Q \text{ units} \times \frac{15.75 / \text{hour}}{60 \text{ minutes/hour}}\right)
\]

\[= 2.625 + 0.722Q\]

Also,

Total cost = average cost/part \(\times\) \(Q\) units

\[= 0.941Q\]

Hence,

\[2.625 + 0.722Q = 0.941Q \Rightarrow 2.625 = 0.219Q \Rightarrow Q \approx 11.986 \rightarrow \text{or about 12 units.}\]
12.11

a. What is the current average labor cost per part, including changeovers?

\[
\text{Cost/part} = \frac{\text{production cost + setup cost}}{\#\text{parts}}
\]

\[
\text{Cost/part} = \frac{[7.5(40) + 55](16.5/60)}{40} = 2.441 \text{ per part}
\]

b. If the changeover time could be reduced to 30 minutes, how much labor cost per part would be saved using the current batch size of 40 units?

\[
\text{Cost/part} = \frac{[7.5(40) + 30](16.5/60)}{40} = 2.269 \text{ per part}
\]

The savings per part is 2.441 – 2.269 = $.172

c. If the changeover time could be reduced to 30 minutes, how much could the batch size be reduced in order to achieve the current average labor cost per unit?

\[
\frac{2.441(X)}{X} = \frac{[7.5(X) + 30](.275)}{X} = 8.25
\]

\[
X = 21.797 \text{ or roughly } 22 \text{ units per batch}
\]
12.12 a. What is the current average labor cost per part, including changeovers?

\[
\text{Cost/part} = \frac{\text{production cost} + \text{setup cost}}{\#\text{parts}}
\]

\[
\text{Cost/part} = \frac{[1.2(60) + 20](14.5/60)}{60}
\]

\[
= \$0.3706 \text{ per part}
\]

b. If the changeover time could be reduced to 10 minutes, how much labor cost per part would be saved using the current batch size of 60 units?

\[
\text{Cost/part} = \frac{[1.2(60) + 10](14.5/60)}{60}
\]

\[
= \$0.3303 \text{ per part}
\]

The savings per part is \$0.3706 – \$0.3303 = \$0.0403

c. If the changeover time could be reduced to 10 minutes, how much could the batch size be reduced in order to achieve the current average labor cost per unit?

\[
\text{Cost/part} = \frac{[1.2(X) + 10](14.5/60)}{X} = \$0.3706
\]

\[
0.3706(X) = [1.2(X) + 10](0.2417)
\]

\[
0.0805(X) = 2.417
\]

\[
X = 30.02 \text{ or about 30 units per batch}
\]

d. What changeover time would be required to produce in batches of 15 units and achieve the current average labor cost per part?

\[
\text{Cost/part} = \frac{[1.2(15) + S](14.5/60)}{15} = \$0.3706
\]

\[
0.01611(S) = 0.0806
\]

\[
S = 5.003 \text{ or about 5 minutes}
\]