Problem 9.9

Without the device (manual operation) – Model 1

Arrival rate: \( \lambda = 50 \) products/hour  
Service rate: \( \mu = 60 \) products/hour  

\[ t_s = \frac{1}{\mu - \lambda} = 0.1 \text{ hour} \equiv 6 \text{ minutes} \]

With the automatic device – Model 2 (constant service times)

Arrival rate: \( \lambda = 50 \) products/hour  
Service rate: \( \mu = 120 \) products/hour  

\[ t_s = t_i + \frac{1}{\mu} = \frac{\lambda}{2\mu(\mu - \lambda)} + \frac{1}{\mu} = 0.00298 + 0.00833 = 0.01131 \text{ hour} \equiv 0.67857 \text{ minute} \]

Time saved per product = \( t_s \) (without device) – \( t_s \) (with device) = 6 - 0.67857 = 5.32143 minutes

Savings per year = Time saved per product (minutes) \times $ saved/minute \times \text{No. of products/year}  
\[ = 5.32143 \times 2 \times 1500 = $15,964 \]

Payback period = \( \frac{\text{Cost of device}}{\text{Savings per year}} = \frac{10,000}{15,964} \approx 0.6264 \text{ year} \approx 7.5 \text{ months} \]

The savings can pay for the cost of the device in roughly 7.5 months!
**Waiting Line Models**

### Problem Name: P9.9 Manual

**Model:** Single Channel

Arrival Rate ($\lambda$) = 50
Service Rate ($\mu$) = 60

Average Number of Units in Waiting Line = 4.1667
Average Number of Units in System = 5.0000
Average Waiting Time in Line = 0.0833
Average Time in System = 0.1000

Probability of Idle System = 0.1667
Probability of 1 unit in the system = 0.1389
Probability of 2 units in the system = 0.1157
Probability of 3 units in the system = 0.0965
Probability of 4 units in the system = 0.0804
Probability of 5 units in the system = 0.0670
Probability of 6 units in the system = 0.0558
Probability of 7 units in the system = 0.0465
Probability of 8 units in the system = 0.0388
Probability of 9 units in the system = 0.0323
Probability of 10 units in the system = 0.0269
Probability of 11 units in the system = 0.0224
Probability of 12 units in the system = 0.0187
Probability of 13 units in the system = 0.0156
Probability of 14 units in the system = 0.0130
Probability of 15 units in the system = 0.0108

### Problem Name: P9.9 Automatic

**Model:** Single Channel with Constant Service Rate

Arrival Rate ($\lambda$) = 50
Service Rate ($\mu$) = 120

Average Number of Units in Waiting Line = 0.1488
Average Number of Units in System = 0.5655
Average Waiting Time in Line = 0.0030
Average Time in System = 0.0113
**Problem 9.12**

Arrival rate: $\lambda = 6$ cars/hour
Service rate: $\mu = 12$ cars/hour

Maximum number of cars in the system (both waiting in line and being washed): $Q = 3$  
*Model 3 applies.*

(a) Proportion of the time system is idle $= 0.5333 \approx 53\%$ of the time

(b) Average number of cars waiting in line $= \bar{n}_q = 0.2857$ car

*** WAITING LINE MODELS ***

PROBLEM NAME: Problem 9.12

MODEL: Single Channel with Limited Waiting Line Length

Arrival Rate (lambda) = 6  
Service Rate (mu) = 12  
Maximum # in System = 3

Average Number of Units in Waiting Line $= 0.2857$
Average Number of Units in System $= 0.7333$

Probability of Idle System $= 0.5333$
Probability of 1 units in the system $= 0.2667$
Probability of 2 units in the system $= 0.1333$
Probability of 3 units in the system $= 0.0667$
Problem 9.14

Arrival rate: \( \lambda = 220 \) customers/hour
Service rate (per server): \( \mu = 35 \) customers/hour
Number of identical servers: \( N = 8 \)  \( \quad \) Model 4 applies.

(c) Proportion of the time system is idle = \( P_0 = 0.0015 \equiv 0.15\% \) of the time

(d) Average time a customer waits in line = \( \bar{t}_l = 0.0071 \) hour \( \equiv 0.426 \) minute

(e) Average number of customers waiting in line = \( \bar{n}_l = 1.567 \)

*** WAITING LINE MODELS ***

PROBLEM NAME: Problem 9.14

MODEL: Multiple Channels

Arrival Rate (lambda) = 220
Service Rate (mu) = 35
Number of Channels = 8

Average Number of Units in Waiting Line = 1.5672
Average Number of Units in System = 7.8529
Average Waiting Time in Line = 0.0071
Average Time in System = 0.0357

Probability of Idle System = 0.0015
Probability of 1 units in the system = 0.0095
Probability of 2 units in the system = 0.0299
Probability of 3 units in the system = 0.0627
Probability of 4 units in the system = 0.0986
Probability of 5 units in the system = 0.1239
Probability of 6 units in the system = 0.1298
Probability of 7 units in the system = 0.1166
Probability of 8 units in the system = 0.0916
Probability of 9 units in the system = 0.0720
Probability of 10 units in the system = 0.0565
Probability of 11 units in the system = 0.0444
Probability of 12 units in the system = 0.0349
Probability of 13 units in the system = 0.0274
Probability of 14 units in the system = 0.0215
Probability of 15 units in the system = 0.0169
Probability of 16 units in the system = 0.0133
Probability of 17 units in the system = 0.0105