EFFECTS OF DETERIORATING INVENTORY ON LOT-SIZING:
A COMPARATIVE ANALYSIS OF HEURISTICS FOR MRP SYSTEMS

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ABSTRACT

This paper considers the effects of deteriorating inventory on lot-sizing in material requirements planning systems. Five existing heuristics, namely nLPC(i), LPC, PPA(–), LTC(–), and LUC, are extended to address the single-level lot-sizing problem with deteriorating inventory and are evaluated via a large-scale simulation study. The simulation study takes three factors into consideration, which are rate of inventory deterioration, percentage of periods with zero demand, and setup cost. Our computational results show that the modified nLPC(i) heuristic possesses the best overall cost performance.

Keywords: Lot Sizing, Material Requirements Planning, Deteriorating Inventory, Empirical Results

1. INTRODUCTION

Material requirements planning (MRP) is a computerized information system for managing dependent demand inventory and scheduling stock replenishment orders. The application of this popular tool in materials management has greatly reduced inventory levels and improved productivity [20]. There exist at least two important issues related to the cost performance of MRP. The first one is planned lead time, the amount of time allowed for orders to flow through the production facility. It plays an important role in the phasing principle of MRP. The second critical issue deals with the question: how to determine how many to order or replenish? This is known as the lot-sizing decision in MRP. The focus of this paper is about the latter of these two issues.

For the single-level uncapacitated lot-sizing problem, Wagner and Whitin [19] introduce a dynamic programming procedure to optimally solve the time varying demand case. Nevertheless, the Wagner-Whitin (WW) algorithm has not been significantly applied in practice [1] [6] because of its considerable mathematical complexity. Haddock and Hubicki [11] conduct a survey of 263 manufacturing companies on the frequency of usage of 11 lot-sizing models in MRP software and conclude that lot-sizing heuristics are applied while the WW algorithm is not used at all. Another important and frequently cited reason for employing heuristics is that heuristics generally outperform WW under the rolling-horizon environment [5], as WW in this environment gives the optimal solution to the wrong problem. Therefore, the vast majority of research has been directed towards developing heuristics to solve the lot-sizing problem [8].
Silver and Meal [18] propose a heuristic, commonly known as Least Period Cost (LPC), that seeks to minimize the average cost of setup and holding per period. Hu and Munson [14] examine ten published numerical studies and find that LPC is the most recommended model. Moreover, they state that LPC is one of two methods which seem to be the most promising heuristics tested under a wide variety of experimental factor settings. Pan [15] performs a detailed sensitivity analysis of six common heuristics using demand pattern, holding cost, and setup cost factors. He concludes that LPC is the best heuristic in producing accurate solutions, as well as being the heuristic most insensitive to uncertainty in parameter estimation. Saydam and Evans [17] provide an extensive computational study on the relative performance of the WW algorithm against four well-known heuristics. Their computational results demonstrate that LPC has the best overall performance among all four heuristics. The average performance of LPC deviates by 1.6% from the optimal cost given by the WW algorithm.

DeMatteis [7] is responsible for the development of the Part-Period Algorithm (PPA). In this exposition, the demand quantity in time \( t \), \( t = i, i + 1, ..., n \), is added to \( Q(i) \) as long as the holding cost in the lot remains less than or equal to the fixed setup cost. An alternative stopping condition used in PPA is to accumulate demands as long as the holding cost in the lot remains strictly less than the setup cost. The alternative version will be labeled as PPA(–), consistent with Baker [2]. Other popular lot-sizing procedures include Least Total Cost (LTC), LTC(–), and Least Unit Cost (LUC) by Gorham [9], Period Order Quantity (POQ) by Berry [3], Economic Order Quantity (EOQ) by Harris [12], and Lot-for-Lot (LFL). Ho et al. [13] recently propose two LPC-based lot-sizing heuristics known as nLPC and nLPC(i) for the single-level uncapacitated case. They perform a simulation study to compare their heuristics with seven existing heuristics, including LPC, and conclude that both nLPC and nLPC(i) yield superior and robust performance under a wide range of experimental conditions.

There exists a large volume of research related to deteriorating inventory. Raafat [16] and Goyal and Giri [10] provide excellent reviews of literature on deteriorating inventory models. However, almost all existing models deal with independent demand. Wee and Shum [20] note that the consideration of deteriorating inventory in MRP systems is almost non-existent and introduce an approach to incorporate the impact of deteriorating inventory in a single-level MRP system. According to their analysis, the inclusion of the effects of deteriorating inventory significantly affects the total relevant cost and the ordering policies and should, therefore, be considered in MRP systems.

In this paper, we study the cost performance of five existing heuristics for the single-level uncapacitated dynamic lot-sizing problem, appropriately modified to address deteriorating inventory. The five existing heuristics are first revised to address the issue of decaying inventory. A large-scale simulation experiment involving three factors, 100 experimental conditions, and 100,000 randomly generated problems is performed to evaluate the five heuristics as modified. Our computational results show that nLPC(i) possesses the best cost performance.

The rest of this paper is organized as follows. The next section discusses the modifications introduced to the five heuristics under consideration. In Section 3, we describe the design of our
2. HEURISTICS FOR DETERIORATING INVENTORY

Deteriorating inventory items are generally divided into two categories. First of all, there exist circumstances where all remaining inventory items become obsolete at the end of the planning horizon. Second, there exist those circumstances where the items deteriorate throughout their planning horizon. This paper focuses on the second type of inventory deterioration.

Wee and Shum [20] discuss the total relevant cost function for the dynamic lot-sizing problem with deteriorating inventory and modify the LPC and LUC heuristics to account for deteriorating inventory. The total relevant cost for a lot equals the sum of the ordering cost, the holding cost in the lot (HC), and the deterioration cost in the lot (DC). In this Section, we discuss how nLPC(i), PPA(–), and LTC(–) may be modified to take the issue of deterioration into consideration. The alternative versions of PPA and LTC, i.e., PPA(–) and LTC(–), are chosen because previous research [2] [12] has shown that they are more effective than PPA and LTC, respectively. The following notation is defined for use in our analysis.

\[ \begin{align*}
S & \text{ fixed setup or ordering cost} \\
h & \text{ unit holding cost per period} \\
\theta & \text{ deterioration rate per period} \\
P & \text{ item unit cost} \\
d(i) & \text{ requirement in period } i \text{ of an } n \text{-period planning horizon} \\
Q(i) & \text{ lot-sizing quantity in period } i \\
TRC(i, j) & \text{ total relevant cost for } Q(i), \text{ where } Q(i) \text{ is intended to cover demands in periods } i \text{ through } j \\
\end{align*} \]

\[ TRC(i, j) \] is computed as follows:

\[ TRC(i, j) = S + h \cdot \sum_{x=i+1}^{j} d(x)/(1-\theta)^{y-i+1} + P \cdot \sum_{x=i+1}^{j} (d(x)/(1-\theta)^{x-i} - d(x)) \quad \text{for } j > i \]

\[ TRC(i, j) = S \quad \text{for } j = i \]

Next, we define modified net average period cost, denoted by mnAPC(i, j), as the ratio of total relevant cost to the number of non-zero demand period(s). Therefore,

\[ mnAPC(i, j) = TRC(i, j)/(j - i + 1 - z) \]

where \( z \) is the number of zero demand period(s) between periods \( i \) and \( j \), inclusive.

The modified nLPC(i) heuristic is given below.

Step 0: Initialize \( i = 1, j = 1, \) \( mnAPC(i, j) = S, \) and \( Q(t) = 0, \) for \( t = 1, 2, \ldots, n. \)

Step 1: If \( j = n, \) then set \( Q(i) = \sum_{x=i}^{n} (d(x)/(1-\theta)^{x-i}) \) and go to Step 5; else set \( j = j + 1 \) and enter Step 2.
Step 2: If $d(j) = 0$, then go to Step 1; else enter Step 3.
Step 3: Compute $mnAPC(i, j)$ using Eq. (2) and find the largest $k$, $i \leq k < j$, such that $d(k) > 0$.
If $mnAPC(i, j) > mnAPC(i, k)$ or $mnAPC(i, j) = mnAPC(i, k) = S$, then set $Q(i) = \sum_{x=i}^{j-1}(d(x)/(1-\theta)^{x-i})$, $i = j$, and $mnAPC(i, j) = S$.
Step 4: Go to Step 1.
Step 5: Find $l = \max\{t : t \leq n, d(t) > 0\}$. If $Q(l) = 0$, then go to Step 7; else set $k_2 = l$ and find $k_1 = \max\{t : t < k_2, Q(t) > 0\}$.
Step 6: If $S > h \cdot \sum_{x=k_{1}}^{k_{2}-1}(d(k_{2})/(1-\theta)^{k_{2}-x}) + P \cdot \sum_{x=k_{1}}^{j-1}(d(x)/(1-\theta)^{x-i} - d(x))$, then set $Q(k_{2}) = 0$ and $Q(k_{1}) = Q(k_{1}) + d(k_{2})/(1-\theta)^{k_{2}-k_{1}}$.
Step 7: Output $Q(t)$, for $t = 1, 2, \ldots, n$.

The modification of DeMatteis’s PPA(−) heuristic is described next. The demand quantity in time $t$ ($t = i, i+1, \ldots, n$) is added to $Q(i)$ as long as the sum of the holding cost and the deterioration cost in the lot is strictly less than the fixed setup cost. That is,

$$h \cdot \sum_{x=i+1}^{j-1}(d(x)/(1-\theta)^{x-i} + P \cdot \sum_{x=i+1}^{j-1}(d(x)/(1-\theta)^{x-i} - d(x)) < S$$

where $i$ and $j$ denote the first and the last periods of the order horizon. Consequently, $j - i + 1$ represents the length of the order horizon. According to Eq. (3), the modified PPA(−) heuristic seeks to find the maximum value of $j$ ($j \geq i$) on condition that $HC + DC < S$.

Lastly, we discuss how LTC(−), the alternative version of the LTC heuristic [9], is extended. Both LTC and LTC(−) aim at selecting the last period of the order horizon with the smallest gap between the total holding cost in the lot and the fixed setup cost. In other words, they attempt to balance these cost components as much as possible. This “balancing” feature gave rise to the name Part-Period Balancing (PPB) and its alternative version, PPB(−), but we choose to call them LTC and LTC(−), compatible with Gorham [9] and Baker [2]. The modified LTC(−) heuristic sets the order horizon equal to the number of periods that most closely matches the sum of total holding cost and total deterioration cost with the fixed setup cost over that period. In case two or more consecutive periods show identical difference between these two terms, the modified LTC(−) selects the smaller number of periods. Let

$$\delta = \left|h \cdot \sum_{x=i+1}^{j-1}(d(x)/(1-\theta)^{x-i} + P \cdot \sum_{x=i+1}^{j-1}(d(x)/(1-\theta)^{x-i} - d(x)) - S\right|$$

Therefore, the modified LTC(−) heuristic seeks to find the minimum value of $j$ ($j \geq i$) such that $\delta$ assumes the smallest possible value.

### 3. SIMULATION EXPERIMENT DESIGN

A simulation experiment is conducted to evaluate the relative performance of the five modified heuristics discussed in Section 2. Three factors are employed to ensure that a diverse and representative set of experimental conditions are created. These three factors are: (1) the
inventory deterioration rate $\theta$, (2) the proportion $p$ of periods with zero demand, and (3) the setup cost $S$.

The deterioration rate is set at five levels: $\theta = 0, 0.01, 0.02, 0.03, \text{ and } 0.04$. These five levels are intended to generate a fairly wide array of deterioration effects. The proportion of periods with zero demand is also set at five levels: $p = 30\%, 40\%, 50\%, 60\%, \text{ and } 70\%$. For periods with non-zero demand, the period demand is generated from a discrete uniform distribution between $a = 10$ and $b = 90$, i.e., $DU(10, 90)$. Hence, five distributions of demand are employed in this study. The mean (standard deviation) of each of the five distributions are $35 (30.14), 30 (30.46), 25 (29.97), 20 (28.59), \text{ and } 15 (26.25)$, respectively. The five distributions produce a wide spectrum of coefficients of variation ranging from 0.86 to 1.75.

We set the planning horizon at $n = 28$. Baker [2] points out that test problems longer than 12 periods could yield additional insight. The item unit cost is $P = $100. The setup cost is set at four levels: $S = $160, $320, $480$ and $640$. The holding cost is assumed to be $2 per unit per period ($h = $2). Consequently, the ratio of setup to unit holding cost, $r = S / h$, is set at four levels: $r = 80, 160, 240, \text{ and } 320$. A Lot-Sizing Index (LSI) is originally introduced by Blackburn and Millen [4] and is defined as $r / \bar{d}$, where $\bar{d}$ denotes average demand per period. According to Blackburn and Millen, LSI is pertinent because it determines the time between orders under constant demand conditions. We define a modified Lot-Sizing Index (mLSI) as $(S / (h + e)) / \bar{d}$, where $e = P \cdot \theta$ is the mean unit deterioration cost. These three factors combined yield a large number of combinations of mLSI values ranging from 0.76 to 21.33. Therefore, experimental conditions are set to yield a wide and representative spectrum of mLSI.

Altogether, the three factors $\theta$, $p$, and $S$ produce a total of 100 ($5 \times 5 \times 4$) experimental conditions. For each set of problems, we create 1,000 replications. Therefore, this simulation study involves the generation and solution of 100,000 problems for each of the five modified heuristics, namely, nLPC(i), LPC, PPA(–), LTC(–), and LUC. Lastly, it should be noted that all heuristics have been programmed in Microsoft Fortran running on a Pentium-based computer.

### 4. COMPUTATIONAL RESULTS

The overall computational results are given in Table 1, with the overall mean percentage deviations in all 100 experimental conditions being -1.253%, -0.294%, 0%, 8.172%, and 11.472% for nLPC(i), PPA(–), LPC, LTC(–), and LUC, respectively. Clearly, nLPC(i) delivers the best overall performance among all five heuristics, with LTC(–) and LUC being the worst performers. Moreover, Table 9 reveals that $p$ is the key factor in determining whether the second best heuristic is LPC or PPA(–). When $p$ is small (30% or 40%), LPC outperforms PPA(–); when $p$ is large (50%, 60%, or 70%), the opposite is true. Furthermore, the performance of LPC relative to each of the other four heuristics deteriorates as $p$ increases. The only small exceptions are for LTC(–) and LUC when $p$ increases from 40% to 50%. On the other hand, the relative performance of LPC to each of the other four heuristics improves as $\theta$ increases. The only exceptions are for LTC(–) when $\theta$ increases from 0.02 to 0.03 and from 0.03 to 0.04. Lastly, the relative performance of LPC to each of the other four heuristics deteriorates as $S$ increases. The only minor exception occurs in PPA(–) when $S$ increases from $160$ to $320$. 
5. CONCLUSIONS

There exists a large volume of research related to deteriorating inventory. However, the consideration of deteriorating inventory for dependent demand in MRP systems is almost non-existent. This paper studies the influence of deteriorating inventory on lot-sizing in MRP systems. Using an approach to capture the impact of deteriorating inventory in a single-level MRP system introduced by Wee and Shum (1999), we propose modifications to three existing lot-sizing heuristics, nLPC(i), PPA(–) and LTC(–), to handle inventory with deterioration. An extensive simulation study is performed to evaluate the relative performance of these three heuristics, along with the modified LPC and LUC heuristics discussed in Wee and Shum [20]. We consider three factors in the simulation study: (1) deterioration rate, (2) proportion of periods with zero demand, and (3) setup cost. These three factors combined produce a fairly wide range of 100 simulated experimental conditions. Two thousand replications are created for each experimental condition. The simulation results show that the modified nLPC(i) has the best performance in all 100 experimental conditions, and it is followed by either PPA(–) or LPC depending on the specific experimental condition. A future research avenue is to consider both deteriorating inventory and quantity discount effects on lot-sizing in MRP systems, since these two effects have opposite influence on the number of orders measure. Therefore, it is interesting to study how different heuristics respond to these two effects. Another possible research avenue is to consider the impact of decaying inventory in multi-level MRP systems.

REFERENCES

References available upon request from Johnny C. Ho, ho@utep.edu.