Course: Math 1332 Contemporary Mathematics.
Class: Lecture Notes. The Mathematics of Finances.

Compound Interest

Compound interest is interest calculated not only on the original principal, but also on any interest that has already been earned. The frequency with which the interest is compounded is called the **compounding period.**

Example I – Calculate Future Value
You deposit $500 in an account earning 6% interest, compounded semiannually. How much is in the account at the end of 1 year?

Solution: The interest is compounded every 6 months. Calculate the amount in the account after the first 6 months. t= $\frac{6}{12}$
A = P (1 + rt)
A = $515[1 + 0.06\left(\frac{6}{12}\right)]$
A = 515
Calculate the amount in the account after the second 6 months.
A = P (1 + rt)
A= $515\ [1 + 0.06\left(\frac{6}{12}\right)]$
A = 530.45
The total amount in the account at the end of 1 year is $530.45.

In calculations that involve compound interest, the sum of the principal and the interest that has been added to it is called the **compound amount**.

The compound amount formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$
where **A is the compound amount**, **P is the amount of money deposited**, **r** is the annual **interest rate**, **n** is the **number of compounding periods per year**, and t is the number of years.

Example II – Calculate the Compound Amount

Calculate the compound amount when $10,000 is deposited in an account earning 8% interest, compounded semiannually, for 4 years.

Solution: Use the compound amount formula.
P = 10,000, r = 8% = 0.08, n = 2, t = 4
$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$A = 10{,}000\left(1 + \frac{0.08}{2}\right)^{2(4)}$

A= 10,000 $(1 + 0.04)^8$
A= 10,000 (1.368569)
A ≈ 10,000(1.368569)
A ≈ 13,685.6

The compound amount after 4 years is approximately $13,685.69

Present Value

The present value of an investment is the original principal (amount of money) invested. Present value is used to determine how much money must be invested today in order for an investment to have a specific value at a future date.

Present Value Formula:

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

Where **P is the original principal invested**, **A is the compound amount**, **r** is the **annual interest rate**, **n is the number of compounding periods per year**, and **t is the number of years**.

Example III – Calculate Present Value

How much money should be invested in an account that earns 8% interest, compounded quarterly, in order to have $30,000 in 5 years?

Solution: Use the present value formula.

A = 30,000, r = 8% = 0.08, n = 4, t = 5

$$P = \frac{30{,}000}{\left(1+\frac{0.08}{4}\right)^{4(5)}} = \frac{30{,}000}{1.02^{20}}$$

$P \approx 20{,}189.14$

$20,189.14 should be invested in the account in order to have $30,000 in 5 years.

Inflation

Inflation is an economic condition during which there are increases in the costs of goods and services. To calculate the effects of inflation, we use the same procedure we used to calculate compound amount.

Example IV – Calculate the Effect of Inflation on Salary
Suppose your annual salary today is $35,000. You want to know what an equivalent salary will be in 20 years—that is, a salary that will have the same purchasing power. Assume a 6% inflation rate.

Solution: Use the compound amount formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, with
P = 35,000, r = 6% = 0.06, t = 20. The inflation rate is an annual rate, so n = 1.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 35{,}000\left(1+\frac{0.06}{1}\right)^{1(20)}$$

$$A \approx 35{,}000(3.20713547)$$

$$A \approx 112{,}249.74$$

Twenty years from now, you need to earn an annual salary of approximately $112,249.74 in order to have the same purchasing power.

The present value formula can be used to determine the effect of inflation on the future purchasing power of a given amount of money.

Example V – Calculate the Effect of Inflation on Future Purchasing Power
Suppose you purchase an insurance policy in 2015 that will provide you with $250,000 when you retire in 2050. Assuming an annual inflation rate of 8%, what will be the purchasing power of the $250,000 in 2050?

Solution: Use the present value formula. $P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

A = 250,000, r = 8% = 0.08, t = 35. The inflation rate is an annual rate, so n = 1.

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

$$P = \frac{250{,}000}{\left(1+\frac{0.08}{1}\right)^{1(35)}}$$

$$P = \frac{250{,}000}{(1+0.08)^{35}}$$

$$P \approx 16{,}908.64$$

Assuming an annual inflation rate of 8%, the purchasing power of $250,000 will be about $16,908.64 in 2050.

Effective Interest Rate

When interest is compounded, the annual rate of interest is called the nominal rate. The effective rate is the simple interest rate that would yield the same amount of interest after 1 year.

Example V – Calculate the Effective Interest Rate
A credit union offers a certificate of deposit at an annual interest rate of 3%, compounded monthly. Find the effective rate. Round to the nearest hundredth of a percent.

Solution: Use the compound amount formula to find the future value of $100 after 1 year.
P = 100, r = 3% = 0.03, n = 12, t = 1

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 100\left(1+\frac{0.03}{12}\right)^{12(1)}$

$A = 100\,(1.0025)^{12}$
$A \approx 100(1.030415957)$
$A \approx 103.04$

Find the interest earned on the $100.
I = A – P
I = 103.04 – 100
I = 3.04

The effective interest rate is 3.04%.

To compare two investments or loan agreements, we could calculate the effective annual rate of each. However, a shorter method involves comparing the compound amounts of each. Because the value of $\left(1+\frac{r}{n}\right)^{nt}$ is the compound amount of $1, we can compare the value of $\left(1+\frac{r}{n}\right)^{nt}$ for each alternative.

Example VI – Compare Annual Yields
One bank advertises an interest rate of 5.5%, compounded quarterly, on a certificate of deposit. Another bank advertises an interest rate of 5.25%, compounded monthly. Which investment has the higher annual yield?

Solution:
Calculate $\left(1+\frac{r}{n}\right)^{nt}$ for each investment.

$\left(1+\frac{r}{n}\right)^{nt} = \left(1+\frac{0.055}{4}\right)^{4(1)}$
≈ 1.0561448

$\left(1+\frac{r}{n}\right)^{nt} = \left(1+\frac{0.0525}{12}\right)^{12(1)}$
≈ 1.0537819

Compare the two compound amounts. 1.0561448 > 1.0537819

An investment that earns 5.5% compounded quarterly has a higher annual yield than an investment that earns 5.25% compounded monthly.